

The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2016-2017

MECHANICS OF SOLIDS 3

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer FOUR Questions

Only silent, self-contained calculators with a Single-Line Display, or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a dictionary to translate between that language and English provided that neither language is the subject of this examination.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

In this examination candidates are required to answer four out of six questions. If a candidate answers more than the required number of questions, all questions will be marked and the highest marks will be used in the final examination mark.

ADDITIONAL MATERIAL: Formula Sheet

INFORMATION FOR INVIGILATOR:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

Turn Over

1. A simply supported beam ABC with a rigid bracket BDE as shown in Figure Q1, is subjected to a vertical load P (equals to 7 kN) at the end of the bracket (point E).

The modulus of elasticity of the beam material, E , is 210 GPa, the second moment of area, I , is $2 \times 10^{-5} \text{ m}^4$ and the total length, L , is 4 m.

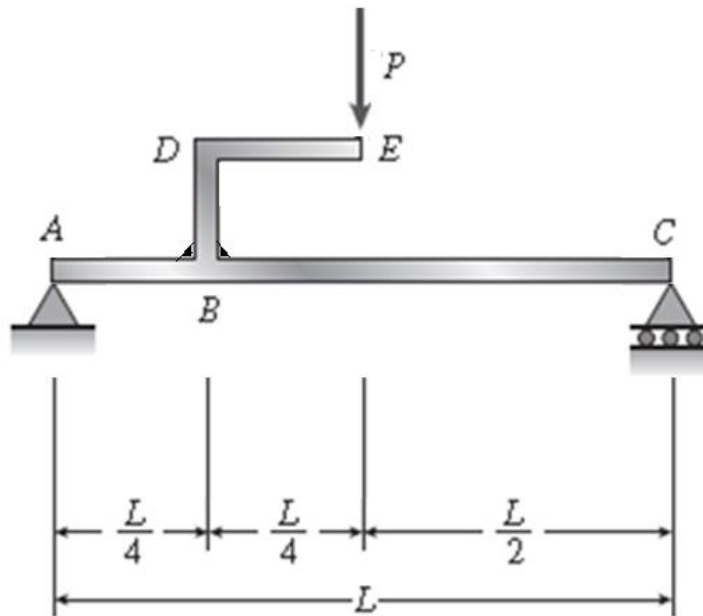


Figure Q1

- (a) Calculate the reactions acting at the supports A and C, determine and sketch the distribution of the bending moment along the beam ABC. [10]
- (b) Calculate the vertical deflection of point B. [15]

2.

The cross-section shown in Figure Q2 is subjected to a bending moment, $M = 125\text{Nm}$, as shown in vector form in Figure 2. The Principal 2nd Moments of Area of the cross-section are:

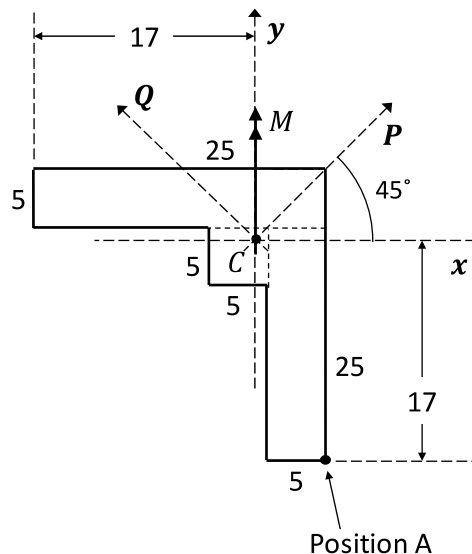
$$I_p = 19,270.83\text{mm}^4$$

$$I_q = 5,395.83\text{mm}^4$$

The principal p-axis is inclined at an angle of 45° anti-clockwise to the x-axis (as shown in the figure).

Determine:

- (a) The angle of the neutral axis with respect to the x-axis and show on a sketch of the cross-section. [7]
- (b) The magnitude of the bending stress at position A. [9]
- (c) The location and magnitude of the maximum compressive bending stress. [9]



All dimensions in mm

Figure Q2

Turn Over

3.

A pair of steel cylinders ($E = 210$ GPa, $\nu = 0.3$) of equal length have the following dimensions:

- i) 100 mm bore and 180.18 mm outside diameter.
- ii) 180 mm bore and 260 mm outside diameter.

The larger cylinder is heated, placed around and allowed to shrink onto the smaller cylinder.

- (a)** Determine the values of Lamé's constants for the problem and state the assumptions made for the stress analysis. [11]
- (b)** Determine the radial and hoop stresses at radii of 50, 70, 90, 110 and 130 mm and plot the distributions on the graph paper provided. [14]

4.

The section shown in Figure Q4 carries a shear force, S , of 48kN, down the vertical centre line.

- (a) Determine the position of the centroid, G , from the base of the section X-X. [4]
- (b) Calculate the second moment of area of the section about the neutral axis (N.A.). [5]
- (c) Determine the vertical shear stresses at points A, B, C, D, E, F and G. [12]
- (d) Sketch the vertical shear stress distribution along the vertical centre line of the section. [4]

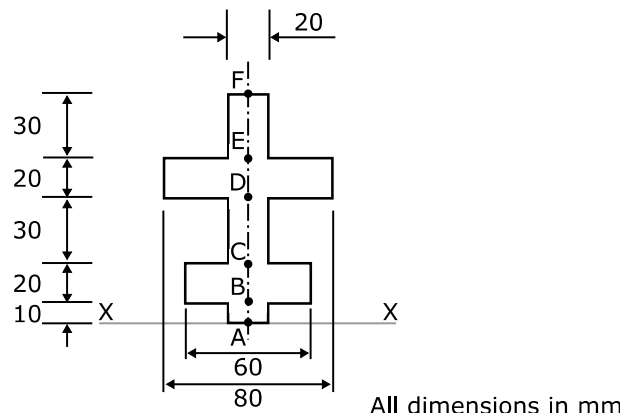


Figure Q4

Turn Over

5.

- (a) Show that the first buckling load, P , for a fixed-free ideal strut (i.e. initially straight with purely axial loading) under compression is given by:

$$P = \frac{\pi^2 EI}{4L^2}$$

where E is the Young's modulus, I is the second moment of area of the cross-section, and L is the length of the strut. Sketch the first buckling mode shape. [8]

- (b) For a strut of length L under compressive loading, define the boundary conditions for the deflection shape for the following end conditions:

i) Fixed-fixed.

ii) Pinned-pinned. [6]

- (c) A rigid beam is supported by a truss pin-jointed structure, as shown in Figure Q5. It is subjected to a point force of magnitude F with direction orientated at 45° to the horizontal plane as shown.

i) Indicate which members are in tension and which are in compression.

ii) Indicate the most critical member due to buckling (justify your answer by calculation). [11]

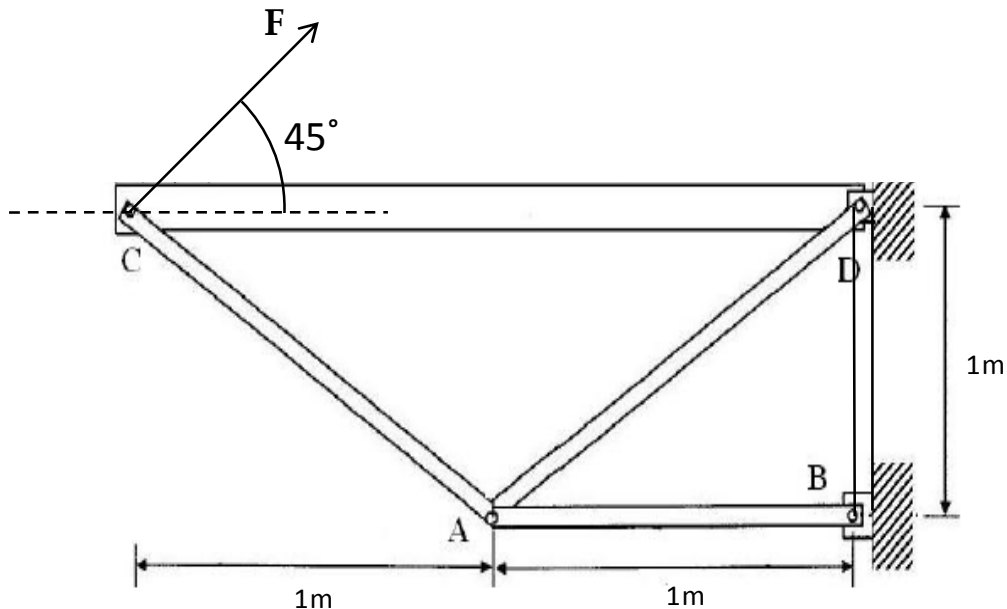


Figure Q5

6. (a) Derive an expression for the strain energy in a uniform section beam of length, L , with Young's Modulus, E , second moment of area, I , subjected to a uniform bending moment, M , along its length. [5]
- (b) The split ring shown in Figure Q6 has a diameter of 400mm and 12mm x 12mm square uniform cross-section. The ring is subjected to equal opposing forces, P , of 125N applied tangentially at the split.

Considering bending strain energy only, determine the amount the split is opened under these forces.

Assume $E = 200\text{GPa}$.

[20]

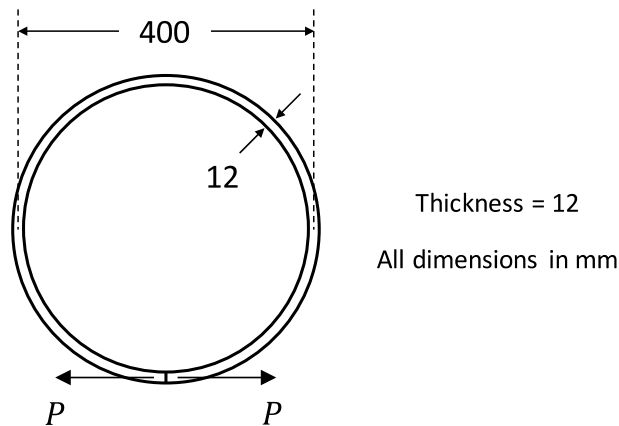


Figure Q6

The following trigonometric identity may be useful when solving for the deflections:

$$\cos^2\phi = \frac{\cos 2\phi + 1}{2}$$

END